

A full Glauber theory for elementary atom – target atom scattering and its approximations

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Abstract

A general formalism of the Glauber theory for elementary atom (EA) – target atom (TA) scattering is developed. A second-order approximation of its full version within the optical-model perturbative approach is considered. The accuracy of the eikonal approximation for potential EA–TA scattering neglecting the excitation effects of TA is evaluated in the framework of second-order Glauber theory for collisions of composite systems.

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1 Introduction

The experiment DIRAC, now under way at PS CERNs [1], aims to observe the relativistic hydrogenlike EA [2]¹ consisting of π^\pm and/or π^\mp/K^\mp mesons (dimesoatoms) and measure their lifetime with a high precision. The interaction of relativistic dimesoatoms with ordinary target atoms is of great importance for this experiment as the accuracy of the EA–TA interaction cross sections is essential for the extraction of the dimesoatoms lifetime.

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¹ Elementary atoms A_{ab} are the Coulomb bound states of two elementary particles. One can enumerate here A_{2e} , $A_{e\mu}$, $A_{2\mu}$, $A_{e\pi}$, $A_{\mu\pi}$, $A_{2\pi}$, $A_{\pi K}$, A_{KK} .

In a series of papers [3] is developed a Coulomb-modified Glauber (eikonal) approximation for the calculations of the total cross sections of the relativistic dimesoatoms interaction with ordinary target atoms. These EA excitation cross sections for the Coulomb interaction with the TA in the eikonal approximation takes into account all multiphoton EA–TA exchange processes.

However, all possible excitations of TA in intermediate and/or finale states in this approximation are completely neglected. In other words, this theory is essentially based on the assumption that the Coulomb potential, created by TA, does not change during the EA–TA interaction. As a result, within this ‘potential’ approximation, the calculated cross sections of the coherent scattering σ_{coh}^{tot} has been identified with the total cross sections σ^{tot} .

The incoherent part of the total cross sections σ_{incoh}^{tot} correspond in the context of the DIRAC experiment to scattering with excitations of the TA electrons from the ground state to all possible excited states. Nuclear excitations of the TA will not be considered here because the much larger excitation energy required (typically on the order of MeV) exceeds the energy range relevant for the application to the dimesoatom–atom scattering [4, 5].

Some estimations of the ratio $\sigma_{incoh}^{tot}/\sigma_{coh}^{tot}$ for the EA–TA scattering were performed in papers [4, 5]. The detailed study of the influence of the target electrons on the EA scattering and the evaluation of the incoherent contributions to the coherent scattering in the first-order Born approximation carry out in [5]. In ref. [6], were announced the simplest results concerning the role of multiphoton exchanges in the incoherent interaction of dimesoatoms with atoms of matter in an approximation which corresponds, as shown further, to the second-order Glauber approximation.

In the present work, the eikonal approximation for potential scattering of EA neglecting all possible excitations of TA [3] has been extended valid for account these effects within the framework of a complete Glauber theory for EA–TA scattering. The paper is organized as follows. In sec. 2, we develop a general formalism of the Glauber theory [7] for the EA–TA interactions. In sec. 3, we consider the second-order perturbation approximation of its full version. Sec. 4 is devoted to the estimation of accuracy of eikonal approximation for potential EA–TA scattering [3] within the second-order Glauber theory for collisions of composite systems. In sec. 5, we reconsider the second-order Glauber approximation in another terms and establish the relationship between the developed formalism and the results of refs. [3]. We also consider results of our analysis in the context of the DIRAC experiment. In conclusion, we short summarize our findings.

We would like to point out that though the theory developed in this work is motivated by concrete experiment, it is also of general interest for high energy physics and atomic physics.

The given work is devoted to memory of my dear friend, the husband, and the co-author, a remarkable human being and scientist Alexander Tarasov, who untimely passed away on March 19th, 2011.

2 Full version of the Glauber theory for EA–TA scattering

A complete Glauber approximation for the amplitude $A_{i+I \rightarrow f+F}(\vec{q})$ of the EA–TA interactions may be given by

$$A_{i+I \rightarrow f+F}(\vec{q}) = \frac{i}{2\pi} \int d^2b \exp(i \vec{q} \vec{b}) \Gamma_{i+I \rightarrow f+F}(\vec{b}), \quad (1)$$

where \vec{q} is the three dimension momentum transverse to the target; the integration is carried out over a plane perpendicular to the direction of incidence; \vec{b} is an impact-parameter vector in this plane; $\Gamma_{i+I \rightarrow f+F}(\vec{b})$ is so-called ‘profile function’.

We may formulate the problem in a general way by considering the EA scattering on a system of Z constituents with the coordinates $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_Z$ ² and the projections on the plane of the impact parameter $\vec{s}_1, \vec{s}_2, \dots, \vec{s}_Z$.

If we introduce the configuration spaces for the EA wave functions $\psi_i(\vec{r})$, $\psi_f(\vec{r})$ and the wave functions $\Psi_I(\{\vec{r}_k\})$, $\Psi_F(\{\vec{r}_k\})$ of the TA constituents in the initial i, I and the final f, F states, we may write the profile function as

$$\begin{aligned} \Gamma_{i+I \rightarrow f+F}(\vec{b}, \{\vec{s}_k\}) &= \int d^3r \psi_f^*(\vec{r}) \psi_i(\vec{r}) \int \prod_{k=1}^Z d^3r_k \Psi_F^*(\{\vec{r}_k\}) \Psi_I(\{\vec{r}_k\}) \\ &\times \Gamma(\vec{b}, \vec{s}, \{\vec{s}_k\}) \end{aligned} \quad (2)$$

with the interaction operator

$$\Gamma(\vec{b}, \vec{s}, \{\vec{s}_k\}) = 1 - \exp[i\Phi(\vec{b}, \vec{s}, \{\vec{s}_k\})] \quad (3)$$

and the phase shift function

$$\Phi(\vec{b}, \vec{s}, \{\vec{s}_k\}) = Z \Delta\chi(\vec{b}, \vec{s}) - \Delta\chi(\vec{b}, \vec{s}, \{\vec{r}_k\}), \quad (4)$$

in which the total phase difference for two EA constituents may be written as a sum of the form

$$\Delta\chi(\vec{b}, \vec{s}, \{\vec{r}_k\}) = \sum_{k=1}^Z \Delta\chi(\vec{b} - \vec{s}_k, \vec{s}), \quad (5)$$

² For the energy range relevant to the dimesoatom–atom scattering, \vec{r}_k ($k = \overline{1, Z}$) is the position vector of an electron in the TA.

and the simple phase difference $\Delta\chi(\vec{b}, \vec{s})$ in (4) one can represent as follows:

$$\Delta\chi(\vec{b}) = \frac{\alpha}{\beta} \int_{-\infty}^{\infty} dz \left[|\vec{R} + \vec{r}/2|^{-1} - |\vec{R} - \vec{r}/2|^{-1} \right], \quad (6)$$

$$\vec{R} = (\vec{b}, z), \quad \vec{r} = (\vec{s}, z), \quad \vec{r}_k = (\vec{s}_k, z_k). \quad (7)$$

Here, Z denotes the TA nuclear charge, α is the fine structure constant, $\beta = v/c = 1$, v is the velocity of the scattering EA in the lab frame, z is the direction of incidence; \vec{R} is the radius vector from the center mass of the target atom to the center mass of EA, \vec{r} is the radius vector from one EA constituent to another.

The amplitude $A_{i+I \rightarrow f+F}(\vec{q})$ (1) is normalized by the relations:

$$4\pi \Im A_{i+I \rightarrow i+I}(0) = \sigma^{tot}(i, I), \quad |A_{i+I \rightarrow f+F}(\vec{q})|^2 = d\sigma_{i+I \rightarrow f+F}/dq_{\perp}, \quad (8)$$

where

$$\sigma^{tot}(i, I) = \sigma_{coh}^{tot}(i, I) + \sigma_{incoh}^{tot}(i, I), \quad (9)$$

$$\sigma_{coh}^{tot}(i, I) = \sum_f \sigma_{i+I \rightarrow f+F}, \quad \sigma_{incoh}^{tot}(i, I) = \sum_f \sum_{F \neq I} \sigma_{i+I \rightarrow f+F}, \quad (10)$$

$$\sigma_{i+I \rightarrow f+F} = \int d^2q \, d\sigma_{iI \rightarrow fF}/dq_{\perp}. \quad (11)$$

To find the total cross sections for all types of collisions in which EA and TA begins in the states i, I , we must sum (10) over all states f, F . The summation is easily carried out by using the completeness relations:

$$\sum_f \psi_f(\vec{r}) \psi_f^*(\vec{r}') = \delta(\vec{r} - \vec{r}'), \quad (12)$$

$$\sum_F \Psi_F(\{\vec{r}_k\}) \Psi_F^*(\{\vec{r}_k\}) = \prod_{k=1}^Z \delta(\vec{r}_k - \vec{r}'_k). \quad (13)$$

Taking into account the expression

$$\begin{aligned} & \sum_{f,F} \frac{1}{2\pi} \int d^2q_1 A_{i_1+I_1 \rightarrow f+F}(\vec{q}_1) A_{i_2+I_2 \rightarrow f+F}^*(\vec{q}_1 + \vec{q}) \\ & = -i [A_{i_1+I_1 \rightarrow i_2+I_2}(\vec{q}) - A_{i_2+I_2 \rightarrow i_1+I_1}^*(-\vec{q})] \end{aligned} \quad (14)$$

and abbreviated $\mathbb{S} \equiv \exp[i\Phi]$ so that we find:

$$\sigma^{tot}(i, I) = 2\Re \int d^2b \left\langle 1 - \langle \langle \mathbb{S} \rangle \rangle \right\rangle, \quad (15)$$

$$\sigma_{coh}^{tot}(i, I) = \int d^2b \left\langle 1 - 2\Re \langle \langle \mathbb{S} \rangle \rangle + |\langle \langle \mathbb{S} \rangle \rangle|^2 \right\rangle, \quad (16)$$

$$\sigma_{incoh}^{tot}(i, I) = \int d^2b \left\langle 1 - |\langle \langle \mathbb{S} \rangle \rangle|^2 \right\rangle, \quad (17)$$

where the double brackets $\langle \langle \rangle \rangle$ signify that an overage is be taken over all configurations of EA and TA in i-th and I-th states.

In so doing the following expressions for the EA and TA form factors take place:

$$\langle f \rangle = \int d^3r |\psi_i(\vec{r})|^2 f(\vec{r}), \quad (18)$$

$$\langle \langle F \rangle \rangle = \int \prod_{k=1}^Z d^3r_k |\Psi_I(\{\vec{r}_k\})|^2 F(\{\vec{r}_k\}). \quad (19)$$

The relation defining the $\langle \langle \mathbb{S} \rangle \rangle$ can be in abbreviated form written down as

$$\langle \langle \mathbb{S} \rangle \rangle = \exp(i\bar{\Phi}), \quad (20)$$

where $\bar{\Phi}(\vec{b}, \vec{s})$ is an effective ('optical') phase shift function in the optical model of the complete Glauber theory.

3 Second-order approximation of the full Glauber theory

In the so-called optical-model perturbative approximation of the Glauber theory, the expansion which we find for $\bar{\Phi}(\vec{b})$ may be written as

$$\bar{\Phi}(\vec{b}, \vec{s}) = \sum_{n=1}^{\infty} \frac{i^{n-1}}{n!} \Phi_n, \quad (21)$$

where

$$\Phi_1 = \langle \langle \Phi \rangle \rangle, \quad \Phi_2 = \langle \langle (\Phi - \Phi_1)^2 \rangle \rangle, \quad (22)$$

$$\Phi_3 = \langle\langle(\Phi - \Phi_1)^3\rangle\rangle, \quad \Phi_4 = \langle\langle(\Phi - \Phi_1)^4\rangle\rangle - 3\Phi_2^2,$$

$$\dots$$

$$\Phi_n \sim Z \left(\frac{\alpha}{\beta} \right)^n.$$

The first order of the expression for $\bar{\Phi}(\vec{b}, \vec{s})$ is the double average of the phase shift function $\Phi(\vec{b}, \vec{s}, \{\vec{s}_k\})$ over all configurations of EA and TA in i, I -states. The second order term of $\bar{\Phi}(\vec{b}, \vec{s})$ is purely absorptive in character and is equal in order of magnitude to the $Z\alpha^2$.

When in the expansion (21) the sum

$$\bar{\Phi}_{tail}(\vec{b}, \vec{s}) = \sum_{n=3}^{\infty} \frac{i^{n-1}}{n!} \Phi_n \ll 1, \quad (23)$$

it seems natural to neglect the $\bar{\Phi}_{tail}(\vec{b}, \vec{s})$ and consider the following approximation:

$$\bar{\Phi}(\vec{b}, \vec{s}) \approx \Phi_1(\vec{b}, \vec{s}) + \frac{i}{2} \Phi_2(\vec{b}, \vec{s}), \quad (24)$$

with

$$\begin{aligned} \Phi_1(\vec{b}, \vec{s}) &= \frac{\alpha}{\pi\beta} \int \frac{d^2q}{q^2} [Z - F(\vec{q})] \\ &\times \left\{ \exp \left[i\vec{q} \left(\vec{b} + \frac{\vec{s}}{2} \right) \right] - \exp \left[i\vec{q} \left(\vec{b} - \frac{\vec{s}}{2} \right) \right] \right\}, \end{aligned} \quad (25)$$

$$\begin{aligned} \Phi_2(\vec{b}, \vec{s}) &= \left(\frac{\alpha}{\pi\beta} \right)^2 \int \frac{d^2q_1 d^2q_2}{q_1^2 q_2^2} \left[F(\vec{q}_1 - \vec{q}_2) - \frac{1}{Z} F(\vec{q}_1) F(\vec{q}_2) \right] \\ &\times \left\{ \exp \left[i\vec{q} (\vec{b}_1 - \vec{b}_2) \right] \right\} \left\{ \exp \left(\frac{i\vec{q}_1 \vec{s}}{2} \right) - \exp \left(\frac{-i\vec{q}_1 \vec{s}}{2} \right) \right\} \\ &\times \left\{ \exp \left(\frac{-i\vec{q}_2 \vec{s}}{2} \right) - \exp \left(\frac{i\vec{q}_2 \vec{s}}{2} \right) \right\}. \end{aligned} \quad (26)$$

The last term in (24) correspond to incoherent scattering.
Accounting the relations

$$\sigma_{coh}^{tot}(i, I) = \langle \sigma_{coh}^{tot}(\vec{s}) \rangle, \quad \sigma_{incoh}^{tot}(i, I) = \langle \sigma_{incoh}^{tot}(\vec{s}) \rangle, \quad (27)$$

$$\sigma^{tot}(i, I) = \langle \sigma^{tot}(\vec{s}) \rangle,$$

we found for all type of the dipole total cross sections $\sigma_{coh(incoh)}^{tot}(\vec{s})$, depending only from properties of a target matter, the following expressions:

$$\sigma^{tot}(\vec{s}) = 2 \int d^2b (1 - \cos \Phi_1 e^{-\Phi_2/2}), \quad (28)$$

$$\sigma_{coh}^{tot}(\vec{s}) = \int d^2b (1 - 2 \cos \Phi_1 e^{-\Phi_2/2} + e^{-\Phi_2}). \quad (29)$$

$$\sigma_{incoh}^{tot}(\vec{s}) = \int d^2b (1 - e^{-\Phi_2/2}). \quad (30)$$

4 Eikonal approximation for potential EA–TA scattering and its accuracy

The eikonal approximation for potential EA–TA scattering neglecting effects of the intermediate excitations of TA (‘potential’ approximation) [3] one can represent as follows:

$$[\sigma^{tot}(i, I)]_{pot} = [\sigma_{coh}^{tot}(i, I)]_{pot}, \quad [\sigma_{incoh}^{tot}(i, I)]_{pot} = 0. \quad (31)$$

If we define the absolute correction to the ‘potential’ approximation of the Glauber theory as

$$\Delta\sigma_{coh}^{tot}(i, I) \equiv \sigma_{coh}^{tot}(i, I) - [\sigma_{coh}^{tot}(i, I)]_{pot} = \langle \Delta\sigma_{coh}^{tot}(\vec{s}) \rangle, \quad (32)$$

we get in terms of total dipole cross sections within the second-order perturbation theory the following expression for its calculation:

$$\begin{aligned} \Delta\sigma_{coh}^{tot}(\vec{s}) &= \sigma_{coh}^{tot}(\vec{s}) - [\sigma_{coh}^{tot}(\vec{s})]_{pot} \\ &= \int d^2b (e^{-\Phi_2} - 1 + 2(1 - \cos \Phi_1)e^{-\Phi_2/2}). \end{aligned} \quad (33)$$

To estimate this correction, we use the bellow-mentioned evaluation formulae:

$$\int \Phi_1^2(\vec{b}, \vec{s}) d^2b \sim (Z\alpha)^2 s^2 L, \quad \int \Phi_1^{2k}(\vec{b}, \vec{s}) d^2b \sim (Z\alpha)^{2k} s^2, \quad (34)$$

$$\int \Phi_2(\vec{b}, \vec{s}) d^2b \sim (Z\alpha^2) s^2 L, \quad \int \Phi_2^2(\vec{b}, \vec{s}) d^2b \sim (Z\alpha^2)^2 \frac{s^4}{R_+^2} L^2, \quad (35)$$

$$\int \Phi_1^2(\vec{b}, \vec{s}) \Phi_2(\vec{b}, \vec{s}) d^2b \sim (Z^3\alpha^4) \frac{s^4}{R_+^2} L^2, \quad (36)$$

$$\int \Phi_1^{2k}(\vec{b}, \vec{s}) \Phi_2(\vec{b}, \vec{s}) d^2b \sim (Z\alpha)^{2k} (Z\alpha^2) \frac{s^4}{R_+^2} L, \quad (37)$$

where

$$L = \ln \frac{R_+^2}{s^2}, \quad \vec{R}_+ = \vec{R} + \frac{\vec{r}}{2}, \quad k \geq 1. \quad (38)$$

Using also the definition

$$\bar{L} = \ln \frac{R_+^2}{\langle s^2 \rangle}, \quad (39)$$

we finally obtain for the relative correction to the dipole total cross sections of the coherent scattering the following estimation:

$$\frac{\Delta \sigma_{coh}^{tot}(\vec{s})}{\sigma_{coh}^{tot}(\vec{s})} = \frac{\sigma_{coh}^{tot}(\vec{s}) - [\sigma_{coh}^{tot}(\vec{s})]_{pot}}{\sigma_{coh}^{tot}(\vec{s})} = O\left(Z\alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \bar{L}\right). \quad (40)$$

We also obtain for the cross sections of elastic scattering

$$\sigma_{i+I \rightarrow i+I}^{el}(i, I) = \int d^2q |A_{i+I \rightarrow i+I}(\vec{q})|^2 \quad (41)$$

with

$$A_{i+I \rightarrow i+I}(\vec{q}) = \frac{i}{4\pi} S_1(\vec{q}) [\sigma_{coh}^{tot}(i, I) + \sigma_{incoh}^{tot}(i, I) S_2(\vec{q})], \quad (42)$$

$$S_2(\vec{q}) \approx \frac{1}{Z} F(\vec{q}), \quad (43)$$

the following relation to its potential approximation:

$$\sigma_{i+I \rightarrow i+I}^{el} = [\sigma_{i+I \rightarrow i+I}^{el}]_{pot} \left(1 + \frac{1}{Z} \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right). \quad (44)$$

Similarly, for the total cross sections of EA inelastic scattering we get:

$$\sigma_{i+I \rightarrow f+I}^{inel} = [\sigma_{i+I \rightarrow f+I}^{inel}]_{pot} \left(1 + \frac{1}{Z} \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right). \quad (45)$$

The relative correction to the potential approximation for the elastic and inelastic cross sections can be then estimated by:

$$\frac{\Delta \sigma_{i+I \rightarrow i+I}^{el}}{[\sigma_{i+I \rightarrow i+I}^{el}]_{pot}} = \frac{\Delta \sigma_{i+I \rightarrow f+I}^{inel}}{[\sigma_{i+I \rightarrow f+I}^{inel}]_{pot}} = \frac{1}{Z} \frac{\langle s^2 \rangle}{R_+^2} \bar{L}. \quad (46)$$

Finally, we find the following relation between the total cross sections of incoherent scattering in the Glauber and Born approximations:

$$\sigma_{incoh}^{tot} = [\sigma_{incoh}^{tot}]_{Born} \left[1 + O \left(Z \alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right) \right], \quad (47)$$

where

$$[\sigma_{incoh}^{tot}]_{Born} = \left\langle \int d^2b \Phi_2 \right\rangle. \quad (48)$$

The difference between the first-order and the second-order total cross section of the incoherent scattering normalized to the first-order cross section, in agreement with [6], is given by

$$\frac{\Delta \sigma_{incoh}^{tot}}{[\sigma_{incoh}^{tot}]_{Born}} = O \left(Z \alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \bar{L} \right). \quad (49)$$

5 Another formulation for the second-order Glauber approximation

To establish a connection between the results of the present work and the refs. [3], we rewrite the simplest results concerning the total cross sections of the EA-TA interactions

$$\sigma^{tot} = \sigma_{coh}^{tot} + \sigma_{incoh}^{tot} \quad (50)$$

in terms of the interaction operators $\Gamma_{coh(incoh)}(\vec{b}, \vec{s})$, according to [6]:

$$\sigma_{coh(incoh)}^{tot} = \int d^3r |\Psi_{i(I)}(\vec{r})|^2 d^2b \Gamma_{coh(incoh)}(\vec{b}, \vec{s}), \quad (51)$$

where $\sigma_{coh(incoh)}^{tot}$ are the total cross sections of EA-TA interaction without or with excitation of the target atom.

In (51) we used the abbreviation $\int \prod_{k=1}^Z d^3r_k |\Psi_I(\{\vec{r}_k\})|^2 = \int d^3r |\Psi_I(\vec{r})|^2$ and operators who are looking like:

$$\Gamma_{coh}(\vec{b}, \vec{s}) = 1 - 2 \cos [\Delta\chi(\vec{b}, \vec{s})] \exp [-\Phi(\vec{b}, \vec{s})/2] + \exp [-\Phi(\vec{b}, \vec{s})], \quad (52)$$

$$\Gamma_{incoh}(\vec{b}, \vec{s}) = 1 - \exp [-\Phi(\vec{b}, \vec{s})]. \quad (53)$$

For $\Delta\chi(\vec{b}, \vec{s}) = \Phi_1$ and $\Phi(\vec{b}, \vec{s}) = \Phi_2$ in eqs. (52), (53) have a place the following expressions:

$$\Delta\chi(\vec{b}, \vec{s}) = \frac{2Z\alpha}{\beta} \int \frac{d^2q}{q^2} \left(e^{i\vec{q}\vec{b}_+} - e^{i\vec{q}\vec{b}_-} \right) [1 - S_1(\vec{q})], \quad (54)$$

$$\begin{aligned} \Phi(\vec{b}, \vec{s}) = \frac{4Z\alpha^2}{\beta^2} \int \frac{d^2q_1}{q_1^2} \frac{d^2q_2}{q_2^2} \left(e^{i\vec{q}_1\vec{b}_+} - e^{i\vec{q}_1\vec{b}_-} \right) \left(e^{-i\vec{q}_2\vec{b}_+} - e^{-i\vec{q}_2\vec{b}_-} \right) \\ \times W(\vec{q}_1, \vec{q}_2), \end{aligned} \quad (55)$$

where $\vec{b}_\pm = \vec{b} \pm \vec{s}/2$, and the function $W(\vec{q}_1, \vec{q}_2)$ is characterized by the equations:

$$\begin{aligned} W(\vec{q}_1, \vec{q}_2) &= S_1(\vec{q}_1 - \vec{q}_2) - S_1(\vec{q}_1)S_1(\vec{q}_2) + (Z - 1) \\ &\times [S_2(\vec{q}_1, \vec{q}_2) - S_1(\vec{q}_1)S_1(\vec{q}_2)], \end{aligned} \quad (56)$$

$$W(\vec{q}, \vec{q}) = S_{incoh}(\vec{q}). \quad (57)$$

The form factor $S_1(\vec{q})$ in (54) can be given by

$$S_1(\vec{q}) = \int d^3r e^{i\vec{q}\vec{r}} \rho_1(\vec{r}), \quad \int d^3r \rho_1(\vec{r}) = 1, \quad (58)$$

and $S_2(\vec{q}_1, \vec{q}_2)$ in eq. (56) can be calculated through the formula

$$S_2(\vec{q}_1, \vec{q}_2) = \int d^3r_1 d^3r_2 e^{i\vec{q}_1\vec{r}_1 - i\vec{q}_2\vec{r}_2} \rho_2(\vec{r}_1, \vec{r}_2), \quad (59)$$

in which

$$\int d^3r_2 \rho_2(\vec{r}_1, \vec{r}_2) = \rho_1(\vec{r}_1), \quad (60)$$

and ρ_1, ρ_2 are the one-particle and two-particle electron densities of the target atom.

The function $\Phi(\vec{b}, \vec{s})$ in the above equations accounts the target atom excitation both in the intermediate and in the final states. If one put $\Phi = 0$, then eqs. (50)-(53) turns to the corresponding relations of refs. [3]. In particular, in this limit $\sigma_{incoh} = 0$.

From (36) it follows that a full relative corrections to $\sigma_{coh}^{tot}(i, I)$ caused by including the multiphoton EA-TA exchange effects, on the one hand, and the intermediate incoherent effects, on the other hand, are of order $Z^3\alpha^4 \ll 1$, and can be successfully neglected. The same is also true for all partial coherent cross sections. These result indicate that the theory of refs. [3] provides quite accurate description of the coherent sector of EA-TA interactions.

As for incoherent interactions, it follows from eq. (49) that they can be described by the Born approximation with the relative accuracy of order $Z\alpha^2$. More precisely, they can be presented in the form:

$$Z\alpha^2 \frac{\langle s^2 \rangle}{R_+^2} \ln \frac{R_+^2}{\langle s^2 \rangle} \quad (61)$$

or, which is equivalent, in terms of radii of the interacting objects:

$$Z\alpha^2 \frac{\langle r^2 \rangle_{EA}}{\langle r^2 \rangle_{TA}} \ln \frac{\langle r^2 \rangle_{TA}}{\langle r^2 \rangle_{EA}}, \quad (62)$$

where $\langle r^2 \rangle_{EA}$ and $\langle r^2 \rangle_{TA}$ are the average constituent-to-constituent distances in EA and TA, correspondingly.

The results of the performed analysis can be summarized as follows: (i) for the description of the coherent interactions of EA with TA it is enough to use the simplified version of the Glauber theory neglecting effects of the intermediate excitations of TA; (ii) for the description of the incoherent interactions of EA with TA it is enough to use the Born approximation. Just such prescriptions based on an intuitive consideration have been proposed by authors of [5].

6 Conclusion

In the present work the simplified Coulomb-modified Glauber approximation for potential EA – TA scattering neglecting the excitation effects of TA [3] is extended to account these effects within the second-order optical-model perturbative approximation of the Glauber theory for collisions of composite systems, according to simplest results of [6]. The complete nonperturbative version of the Glauber theory for EA – TA scattering accounting all possible excitations of EA and TA in intermediate and/or finale states is formulated.

References

- [1] B. Adeva *et al.*, *Lifetime measurement of $\pi^+\pi^-$ atoms to test low energy QCD predictions* (Proposal to the SPSLC, CERN/SPSLC 95–1, SPSLC/P 284, Geneva), 1995;
B. Adeva, L. Afanasyev, M. Benayoun *et al.*, *Phys. Lett.* **B619** (2005) 50;
L. Afanasyev, A. Dudarev, O. Gorchakov *et al.*, *Phys. Lett.* **B674** (2010) 11;
B. Adeva, L. Afanasyev, M. Benayoun *et al.*, *arXiv:hep-ex/1109.0569*;
see also <http://www.cern.ch/DIRAC/>
- [2] L.L. Nemenov, *Yad. Fiz.* **15** (1972) 1047; *ibid.* **16** (1972) 125; *ibid.* **41** (1985) 980;
S. Mrówczyński, *Phys. Rev.* **D36** (1987) 1520; K.G. Denisenko and S. Mrówczyński, *ibid.* **D36** (1987) 1529;
S.H. Aronson *et al.*, *Phys. Rev. Lett.* **48** (1982) 1078; J. Kapusta and A. Mocsy, *arXiv:nucl-th/9812013*.
- [3] A.V. Tarasov and I.U. Christova, JINR Communication P2-91-10, Dubna, 1991;
S.R. Gevorkyan, A.V. Tarasov, and O.O. Voskresenskaya, *Phys. At. Nuclei* **61** (1998) 1517;
L. Afanasyev, A. Tarasov, and O. Voskresenskaya, *J. Phys.* **G25** (1999) B7;
M. Schumann, T. Heim, K. Henken *et al.*, *J. Phys.* **B35** (2002) 2683.
- [4] A.S. Pak and A.V. Tarasov, JINR-E2-85-882, Dubna, 1985;
A.S. Pak and A.V. Tarasov, JINR-E2-85-903, Dubna, 1985.
- [5] T.A. Heim, K. Henken, D. Trautmann *et al.*, *J. Phys.* **B33** (2000) 3583.
- [6] A. Tarasov and O. Voskresenskaya, in: *Proc. Int. Workshop on Hadronic Atoms, “HadAtom02”*, *ArXiv:hep-ph/0301266*.
- [7] R.J. Glauber, *Phys. Rev.* **100** (1955) 242;
R.J. Glauber, in: *Lectures in Theoretical Physics*, v.1, ed. W. Brittain and L.G. Dunham. Interscience Publ., N.Y., 1959, 315 p.;
V. Franco and R.J. Glauber, *Phys. Rev.* **142** (1966) 1195;
R.J. Glauber, in: *High Energy Physics and Nuclear Structure*, ed. G. Alexander, North-Holland Publ. Company, Amsterdam, 1967, 310 p.
R.J. Glauber, *Uspekhi Fiz. Nauk* **103** (1971) 641.